is true of x_2 and x_2'. Hence, ∞ is a root of the sextic covariant, whose leading coefficient must therefore vanish. This gives $8a_0^2a_3 - 4a_0a_1a_2 + a_1^3 = 0$ as the required condition. The same result may be readily obtained by elementary methods.

Also solved by HERBERT N. CARLETON, OTTO J. RAMLER, and HORACE OLSON.

E. B. ESCOTT sent in three solutions of 515 (Geometry). The first two solutions were solutions given in E. CATALAN'S *Théorèmes et Problèmes de Géométrie Élémentaire,* pp. 237–239. The third is a neat original solution.

516 (Geometry). Proposed by R. M. MATHEWS, Riverside, California.

Through the edges of a trihedral angle planes are passed orthogonal to the opposite faces Prove them coaxial.

I. SOLUTION BY NATHAN ALTSHILLER, University of Oklahoma.

The planes x, y, z which pass through the edges OA, OB, OC of the trihedral angle O–ABC and which are perpendicular to the opposite faces a, b, c, cut these faces along lines OD, OE, OF, respectively. A plane p perpendicular to OA cuts the edges OA, OB, OC in the points A, B, C, and the lines OD, OE, OF in the points D, E, F. (The reader may readily construct the figure.)

The plane x is perpendicular to a by construction, and to the plane p, because x passes through OA; hence, x is perpendicular to the line of intersection BC of p with a, and therefore BC is perpendicular to AD.

The plane b is perpendicular to y by construction and to the plane p, because b passes through OA; hence, b is perpendicular to the line of intersection BE of y with p, and, therefore, BE is perpendicular to AC. For similar reasons, CF is perpendicular to AB.

The three altitudes AD, BE, CF of the triangle ABC concur, according to a well-known proposition, in a point H, the orthocenter of ABC; hence H is a common point of the planes x, y, z. Now these three planes have obviously the point O in common, hence they pass through the line OH.

Incidentally we have also proved: *The locus of the orthocenter of the triangle determined by three concurrent lines and a variable plane perpendicular to one of the given lines is a straight line concurrent with the given lines.*

II. SOLUTION BY W. WOOLSEY JOHNSON, Annapolis, Md.

Referred to the sphere, the problem is that of the existence of the orthocenter of the spherical triangle ABC.

Let $CD = p$ be the perpendicular from C upon AB and let the perpendicular from A upon BC cut it in O.

From the right triangle AOD we have

$$\tan OD = \tan OAD \sin AD. \tag{1}$$

From AEB, we have

$$\cot OAD = \cos c \tan B. \tag{2}$$

From ADC, we have

$$\sin AD = \tan p \cot A. \tag{3}$$

Dividing (3) by (2) and substituting in (1), we have $\tan OD = \tan p \cot A \cot B \sec c$.

Interchanging A and B, c and p remain unchanged; hence, the perpendicular from B cuts off from p the same segment OD or the three perpendiculars meet in a common point.

Also solved by HORACE OLSON, L. E. LUNN, C. J. PAYNE, WILLIAM HOOVER, and the PROPOSER.